Research about Technology in Mathematics Education: an evolution.

J.B. Lagrange

IUFM Reims & Laboratoire de Didactique des Mathématiques, Université Paris 7.

jb.lagrange@reims.iufm.fr

Research about Technology in Mathematics Education is a very active field offering a wealth of stimulating ideas, experiments and theorisations. The context of this research is the abundance of technologies, the quick pace of their development and the strong pressure of society contrasting with slow progressing actual classroom uses of computer tools.

The evolution of research about Technology in Mathematics Education can be seen as an effort to account for this contrast, offering visions for the future, but also considering the concrete conditions of classroom implementation. I will specify this idea by looking at different technologies together with specific theoretical concerns:

- programming with the reification theories,
- microworlds with situated cognition,
- spreadsheets and computer symbolic systems with the instrumental and anthropological approaches,
- today fast developing web based technologies with the need for new approaches.

Programming is an important feature of computer technology and many specific languages have been proposed as means to manipulate mathematical entities. A central assumption has been that programming helps learners to reflect on actions and then favours conceptualisation. Theories have been built to support this assumption. An example of a programming language associated with a specific theory is Dubinski 's ISETL language and APOS theory (Czarnocha and al., 1999). Dubinski 's approach sees conceptualisation as a construction: the learner first considers actions (executed in command mode), then 'compiles' actions into processes (corresponding to computer procedures) and finally 'encapsulates' processes into new objects (new objects are added to the programming language). Actions can then be again considered on these new objects.

This view was challenged by theories about visualisation (Tall 1999) that proposed to take advantage of the multiple representations of mathematical entities allowed by computer technologies to favour more flexible approaches to conceptualisation.

The idea of micro-world was to offer learners a more or less virtual space where he (she) could freely conceptualise by considering questions and constructing solutions. This idea was powerful enough as to evolve from the first vision linked to 'turtle geometry' (Papert 1980) to recent projects like MathLab (Noss & Hoyles 2006), based on the idea of building new representations. Because computer objects and representations generally differ from usual mathematics, math educators became aware that conceptualisation always depends on situations and questioned the notion of abstraction (Noss & Hoyles 1996), introducing the idea of connection. The importance of communication in social processes of construction also appeared, supported by the development of computer connectivity.

When computer symbolic systems were made available for classroom use, they raised a lot of attention, with the central assumption that they could allow quick and easy actions in problem solving and then favour conceptualisation especially in algebra and calculus by dramatically reducing the part of meaningless technical manipulation. The spreadsheet, and the specific notation it offers to express relationship between entities, appeared to have a great potential for introducing younger students to algebra, preparing them to notions like variables, equations and functions.

Difficulties however happened when implementing tools like computer symbolic systems and spreadsheets in the classroom. It appeared first that, to benefit of the tool's potential, a learner needs knowledge intertwining mathematical understanding and awareness about the tools functioning. Acquiring this knowledge is a non-obvious and time-consuming process, that psychologists like Rabardel (1995) named 'instrumental genesis'.

It also appeared that the relationship between the technical part of the mathematical activity and conceptualisation was not so simple: suppressing paper-pencil techniques also suppressed the possibility of reflection on these, which was useful for conceptualisation. Researchers then became aware of the need for techniques as a basis for a conceptual reflection (the 'epistemic' value of techniques, opposed to their 'pragmatic' value). They started to think of new techniques that computer tools make possible rather than to try to eliminate the technical part of the mathematical activity (Artigue 2002, Lagrange 2005).

Another growing awareness is that, as educators, we cannot simply orchestrate software applications that industry or computer science creates. That is why 'design' is now an important dimension of research. Yerushalmy (1999), for instance, saw a discrepancy between encouraging evidence about the impact of various specific software capabilities and discouraging evidence about work with educational software that does not always act as the idea generator it was designed to be. She asked designers for more work, not just having good ideas, but also to realize and articulate their perhaps unconscious decisions and turn them into conscious design considerations.

Today fast developing web based technologies offer a new challenge to mathematics education. Possible uses are very varied (Engelbrecht and Harding, 2005), from simple on line hypertext resources to full interactive courses. Theoretical work is needed to integrate ideas about communication and social interaction. Other important fields for research are resources' indexation and teachers' proficiencies for a wise use of technology.

References

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning* 7(3), 254-274

Czarnocha, B., Dubinsky, E., Prabhu, V., Vidakovic, D. (1999). One theoretical perspective in undergraduate mathematics education research. Proceedings of PME 23, I, 95–110

Engelbrecht J., Harding A., Teaching Undergraduate Mathematics on the Internet, Educational Studies in Mathematics, Volume 58, Issue 2, Feb 2005, Pages 253 – 276

Lagrange, J.B., (2005). Using symbolic calculators to study mathematics. The case of tasks and techniques. Chapter 5 in Guin D., Trouche L. (eds.), *The integration of symbolic calculators.* Kluwer

Noss R. & Hoyles, C. (2006) Exploring Mathematics through Construction and Collaboration. In K.R. Sawyer (Ed) *Cambridge handbook of the Learning Sciences.* Cambridge: CUP

Noss R. & Hoyles C. (1996) *Windows on Mathematical Meanings: Learning Cultures and Computers.* Dordrecht: Kluwer

Papert, S., 1980, Mindstorm. Children, computers, and powerful ideas- Basic Books, Inc. New York, NY, USA

Rabardel P. (1995), *Les hommes et les technologies - Approche cognitive des instruments contemporains.* Armand Colin, Paris.

Tall, D. O. (1999). Reflections on APOS theory in Elementary and Advanced Mathematical Thinking. In O. Zaslavsky (Ed.), *Proceedings of the 23rd Conference of PME, Haifa, Israel*, 1, 111–118

Yerushalmy, M. (1999). Making exploration visible: on software design and school algebra curriculum, *International Journal of Computers for Mathematical Learning*, vol 4, 169-189