

**REVISITING AN OUTSTANDING RESEARCH PROBLEM IN
MATHEMATICS EDUCATION: THEORY VERSUS PRACTICE. THE CASE OF
TEACHER PROFESSIONAL DEVELOPMENT FROM THE VIEW OF A
MATHEMATICIAN.**

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Many scientific domains are occupied with the central problem how science might inseminate and empower practice and how practice might orientate science. The duality in mathematics: pure mathematics versus applied mathematics is a further prominent example, often questioned and still not solved satisfactory and finally. The same applies to the similar question in mathematics education. The list of contributing authors is long and outstanding. So far, what can be said? Are there any new insights today? The talk will illuminate the various dimensions of this important duality; however, also enlighten the (still) missing bridge buildings. In particular, the field of teacher professional development will be focused on and some personal examples will be given while reflecting a recent theory-practice-intervention in Germany from the view of a mathematician.

The talk is divided into two parts, ironically speaking into a theoretical analysis, where we are question whether it is meaningful to work with the *theory-practice*-dichotomy, and a more practical orientated report where we show that e.g. in professional development this distinction is not satisfactory explaining.

Theory versus practice? Bridging theory and practice! A convincing dichotomy! What might be the intended messages – in a first approach? Firstly, we need both sides for successful mathematics teaching. Accepted! Secondly, if there are deficits in the classroom, then, their might be a gap between both, not enough theory applied, practice not deeply enough theorized. Maybe! Thirdly, there are only two main variables: theory and practice and their coherence will do our educational job!? Hardly to believe!

Nevertheless, this couple of terms has often been used to headline conferences in the past. The titles are broad enough to cover nearly everything showing that this duality is not very precise. Some conferences, e.g., the 2003 PME-NA Annual Meeting in Hawaii, and the Fifth International Conference on Systematic Cooperation between Theory and Practice in Mathematics Education in 1994 declared the two strands as their central topic for the congress. And there are numerous contributions of outstanding fellow colleagues or editorial boards: NCTM (1953), IDM (1976), Davis (1987), Steiner (1987), Scott-Hodgetts & Lerman (1990), Brown & Cooney (1991), Wittmann (1991), Crawford (1992), Ross & et al. (1992), Cooney (1994), Steinbring (1994), Lerman (1994), Schoenfeld (1999), Jeronen & Pikkarainen (1999), Boaler (2000), Malara & Zan (2002), Ziloix (2003), Malara (2003), English (2003), Bussi & Bazzini (2003), Dougherty & Ziloix (2003), Even & Ball (2003), Even & Schwarz (2003), Szendrei (2007) etc. Of course, such a list cannot be exhaustive. The number of contributions emphasizes that the duality is a satisfactory issue for (theoretical) discussion; we personally recommend the recent article of Malara & Zan (2002) and will often refer to this contribution.

The comments of the author insinuate that he is not very happy with the *theory-practice*-dichotomy, the foreground appears convincing at a first sight, however, the background is not! Should this dichotomy not finally be declared as dead? In general, we should be very cautious with these two terms.

Binary explanations are black-white modeling attempts and by no means, the reality in classroom teaching and learning is explainable in binary terms. Binary explanations raise

according to the hen-egg-problem the questions, *Which of them came first? What is more important, what is less significant?* One may think whether one should accept 'theory' and 'practice' as the dual sides of a coin, so far, one can speak of a Janus-faced phenomenon. But then, there is no place for further additions.

One is soon reminded on a similar dichotomy often referred to in the domain of mathematics, namely *the dichotomy of pure and applied mathematics*. Although it is interesting that Schoenfeld (2007) regards mathematics education in some sense as applied mathematics, many research mathematicians articulate that time is long over to think in terms of pure versus applied mathematics. Even 100 years ago, F. Klein (1924, p. 39) proposed to use the better fitting wording of '*precision mathematics*' versus '*approximation mathematics*', however he did not succeed in making this wording popular. It was the famous mathematician P.R. Halmos (1968), who declared *applied mathematics as bad mathematics*. Thus, is theory a clean science and practice, however, a dirty involvement?

It is an irony of history of mathematics that areas in which Halmos did research and which he claimed as typical pure mathematics are of highly importance for modeling applicable mathematics. Thus, there is no clear borderline any longer to distinguish between pure and applied, between theory and reality. And, as the author may prove by examples of his work, doing applied mathematics is quite different from just applying pure mathematics to specific situations in reality.

Next, we want to emphasize that the terms 'theory' and 'practice' are highly philosophically loaded. Whereas answers in mathematics, what might be understood by theory, what might be understood by reality, can be answered satisfactory, theory in mathematics education is of quite different nature than in mathematics, where nearly every axiomatic framework might be worthwhile to be studied in detail. Again, there is a vast list of contributions to this aspect, e.g., Malara & Zan (2002) point out different strands and provide some satisfactory and interesting answers. Further main streams of this discussion will be presented in the talk.

In the second section, in the practice-orientated part, the author will discuss some subjective as well as personal experiences when starting an intervention project financed by some foundation with the objective to open new approaches to in-service teacher education and moreover to promote teacher professional development.

As we started our aforementioned project 'Mathematics done differently', we soon have stumbled over various obstacles – on the side of theory as well as on the side of practice. A few of them are listed below, others will be addressed in the talk in more details:

(i) *The problematic issue of the relevant theory or theories*: to identify and to find the adequate theory (ies) is like choosing an adequate lens to understand the situations. In the classroom situation plenty of variables can be observed, however, only a handful of them can be controlled. To choose the 'right' variables is a subjective decision by the researcher. Thus, it is not only a question of finding and applying but to parameterize situations, to adopt theory elements and to specify or metaphorically speaking to domesticate theory. Or to quote Karl Popper (1935, p. 26) 'theory is the net which we throw out in order to catch the world'.

(ii) *The delicate role of consulting various experts*: Of course, in our situation of designing and redesigning the project we were very open to suggestions and proposals. We had to learn that different experts prefer different theories and give different, sometimes controversial advices – and this should be accepted, however life becomes thereby not easier.

(iii) *The unknown recipients of our intervention initiatives*. Of course, we knew that we have to address teachers, teachers of primary schools as well as teachers of secondary schools. But the equation: teacher = teacher does not hold! Clearly, we want to know as much as possible about our clients. And we had to accept a quotation of Malara & Zan (2002, p. 554): '*This thought-provoking question emphasizes the fact that most studies are about teachers but*

not with and for teachers'. What about teachers' knowledge, beliefs, awareness, and their emotions? In many cases, these are blank spots.

(iv) *The dependence on mediators*. The advisors in our in-service training courses function like 'customer consultants'. Seldom, we interact directly with teachers or students. We are highly dependent on these mediators, but how to calculate them? Of course, we chose the 'best speakers', but this is just an isolated opinion of someone to declare some persons as best fitting to a specific situation. It should be noted that our training courses are run by a tandem of a researcher and a practitioner. Insofar, we try to take care of the *theory-practice*-dichotomy, following again Malara & Zan (2002), by reconciling between theory and practice through collaboration between teachers and researches.

(iv) *The social variables*. The teaching and learning processes enrolled in our courses did not address single persons or individuals but groups of teachers which do have their own social dynamics. Thereby, the challenge was to understand these processes.

(v) *Sustainability*. Of course, only sustainable interventions count. But what is sustainability? It is practice, which defines sustainability, not research!

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