

Presentation of my PhD Research Work in Progress (METU, 2008)

by

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Title: Undergraduate Mathematics Students' Understanding of Proof by Contraposition in Verbal, Symbolic and Visual Contexts with respect to their Learning Styles

Although studies indicating consistent results on student difficulties with indirect proofs that seem to be greater than those related to direct proofs at any grade level, literature indicates that not much attention was devoted to students' understanding of indirect proofs particularly proof by contraposition so far (Antonini&Mariotti, 2007). There exists a couple of studies investigating students' understanding of contraposition principle ($p \rightarrow q \equiv \sim q \rightarrow \sim p$) through conditional statements (Antonini 2004; O'Brien, 1972,1973;Wu Yu et. al 2003).However, relatively few studies are located on students' understanding of proof by contraposition(Antonini&Mariotti,2007;Goetting,1995;Stylianideset.al,2004). Understanding proof by contraposition involves both the procedural knowledge of negating a statement, recognizing the steps required for proof by contraposition and the conceptual knowledge on recognition of contraposition principle. Only one study is located to investigate students' understanding of proof by contraposition in verbal and symbolic contexts across their majors (Stylianides et. al, 2004). However, this addresses a gap in the existing literature since students' understanding of proof by contraposition is not also explored in one of the oldest yet fundamental type of context namely visual context. Existing literature points out that context is found to be an extremely important variable in performance on deductive reasoning tasks (Inglis&Simpson, 2006, 2008; Johnson-Laird et. al, 1972). Since it is an important confounding factor affecting students' reasoning to study the underlying logical behavior of students it would be practical to include as many contextual factors as possible. Moreover, using isomorphic (logically equivalent) tasks eliminates the effect of task structure and facilitates assessing students' performance attributable to specific context. There exist studies indicating that there are variations in students' reasoning depending on the particular context (Inglis&Simpson 2006; Stylianides et. al, 2004). Thus far, the students' understandings of proof by contraposition have remained relatively unexplored in both three contexts (verbal, symbolic and visual) and what is more, it has not been examined with

respect to students' learning styles. The literature suggests the necessity for research in comparing the relationship between students' learning styles, their proving approaches and what they learn in their advanced mathematics courses (Weber, 2005). Yet there is need for qualitative approach to explore underlying meanings during students' justifications of their conclusions and to examine the structure of their argumentations when supporting a conjecture and its proof as well as understanding the logical cognitive connection between statements. To achieve this aim, structural diagram method introduced by Krummheuer (1995) based on Toulmin's (1958) categories will be implemented.

The aim of this study is to investigate undergraduate mathematics students' understanding of proof by contraposition in verbal, symbolic and visual contexts with respect to their learning styles and to analyze students' underlying thinking processes upon (1)negating a statement, (2) recognizing the procedure that is required for proof by contraposition and (3) recognizing the contraposition principle in verbal, symbolic and visual contexts with respect to their learning styles through a structural diagram method introduced by Krummheuer (1995) based on Toulmin's (1958) categories.

This study combines both qualitative and quantitative approaches in a sequential explanatory mixed design where quantitative and qualitative data obtained will be integrated during the interpretation phase of the study and qualitative results will be used to assist in explaining and interpreting the findings of the primarily quantitative study.

To accomplish the purpose of this study, the research question for the quantitative part is:

- Are there any significant differences among students with different learning styles in terms of their performances in verbal, symbolic and visual contexts?

And the research questions regarding the qualitative part are:

- How does a student with specific learning style conceptualize negation of a statement in different contexts?
- How does a student with specific learning style conceptualize the procedure required for proof by contraposition in different contexts?
- How does a student with specific learning style conceptualize contraposition principle in different contexts?

This study consists of two theoretical perspectives:

Similar perspective will be implemented that was used by Stylianides et. al (2004) which is the only study that is located to examine students' understanding of proof by contraposition both in verbal and symbolic contexts at the same time with respect to students' majors. However, my current research will contribute to the current literature by investigating students' understanding of proof by contraposition with respect to their *learning styles* and *visual* context will be explored in addition to symbolic and verbal contexts by using *isomorphic* tasks to eliminate the effect of task structure and to facilitate the assessment of students' performances that is attributed to specific context.

Argument theories of Toulmin (1958) provide a theoretical framework for analyzing the structure of the reasoning that occurs during the process of argumentation and to organize the students' justifications. The Toulmin's structural diagrams depict a general outlook of students' reasoning. The content of each diagram is derived from the assumptions or *data*, the *conclusions*, the *warrants*, and the *backing* of student's responses during the interview.

The participants of this study will be all first year undergraduate mathematics students who are enrolled in Calculus courses in public (Ankara, Anadolu, Gazi, Hacettepe, METU (Middle East Technical University)) universities located in Ankara and in Eskişehir.

Currently, the development and checking face and content validity of the task based questionnaire is still in progress. The pilot administration of the task based questionnaire and the Learning Styles Inventory will probably be conducted at 2008 summer school (July, 1-August, 15) with first year mathematics students who are taking Calculus course in METU. After reviewing the questionnaire and the interview protocol depending on the findings of the pilot study, basic study will be conducted in 2008 Autumn Semester with the aforementioned university participants.

In order to gather quantitative data, Kolb (1985)'s Learning Style Inventory which was adopted into Turkish by Aşkar and Akkoyunlu (1993) and a task based questionnaire in Turkish including isomorphic (logically equivalent) tasks in verbal, symbolic and visual contexts will be conducted. The quantitative data from the inventory and the questionnaire

will be supplemented by the semi-structured interviews in order to acquire deeper understanding on students' underlying reasoning. The qualitative data will emerge from semi-structured interviews with students representing each learning style group. The focus points of the interviews are: (1) the conception of negating a statement, (2) recognition of the procedure required for proof by contraposition (3) recognition of contraposition principle. Students' reasoning in the interviews will be transcribed and analyzed by using schematic structural diagrams.

Descriptive statistics such as frequencies, percentages, and cross-break tables concerning students' learning styles across tasks in verbal, symbolic and visual contexts will be obtained to provide detailed information about the distribution of answers. MANOVA will be carried out in the SPSS software to detect possible significant differences among learning style groups on the population mean performance scores in verbal, visual and symbolic contexts. Three dependent variables of the study are the performance scores of students in verbal, symbolic and visual contexts. They are conceptually related and do not exhibit linear dependency on each other. The categorical, independent variable is the learning styles of students with four levels (accommodative, assimilative, convergent and divergent).

Thus far, there exists no research pertinent to the scope of this study. Consequently, this study informs instructors and curriculum developers on students' reasoning and understanding in different contexts by taking into consideration their learning styles, their difficulties related to the context in understanding the proof by contraposition. In addition, results obtained from this study emerge important implications on improvement of teaching regarding learning styles. Some pedagogical implications may appear on what other aspects of proof instructors should point out in their classrooms in order to develop understanding of proof particularly proof by contraposition. The interpretation of the findings will reveal the learning style and context specific considerations of students' understanding of contrapositive principle and highlight the factors that influence students' logical thinking.

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