UNDERGRADUATE STUDENTS' DIFFICULTIES WITH MATHEMATICAL PROOF AND PROOF TEACHING[1]

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Introduction

Proof is the basis of mathematics and it is important for students to know what constitutes a proof, why proof is needed and how to construct proof to understand the structure of mathematics.

The meaning of proof, its role and the way it is created, verified and accepted may vary from person to person and from community to community (Harel&Sowder, 2007). Also the proof aspect of mathematics, its place and extent in the curriculum shows the greatest variation when viewed internationally (Bell, 1976).

NCTM (2000) emphasizes that:

"Reasoning and proof are not special activities reserved for special times or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what the topic is being studied." (cited in Mariotti, 2006).

In Türkiye proof takes place in secondary school geometry lessons at elementary level and in ninth grade mathematics curriculum in the learning field of logic as proof methods. But students first meet with the proof concept as accepted by mathematicians community at university level. In their previous experiences students struggle mostly with the computational aspect of mathematics and see mathematics as a list of facts, rules and procedures. Therefore when they come university level, they have difficulties with understanding the abstract and axiomatic structure of mathematics based on concepts, relations between concepts, definitions, theorems and proofs.

In some universities there are transition to proof or introduction to mathematical reasoning courses (Selden&Selden, 2007a; Knapp, 2005; Baker&Campbell, 2004; Epp, 2003; Smith, 2006), to facilitate students understanding of the formal mathematics language and axiomatic structure. The content of these courses is not standartized but they typically include topics such as logic, sets, and functions, information assumed to be a prerequisite for constructing proofs (Selden&Selden, 2007b; Epp,2003). Many researcher study students' conceptions of proof and difficultes in constructing proof in the context of these courses and found that these courses are not effective in overcoming students' difficulties and facilitating their proof construction processes.

Background of the study

Although proving is very important part of advanced mathematics, studies (e.g. Moore, 1994; Harel&Sowder, 1998; Dreyfus, 1999; Almeida, 2000; Jones, 2000; Weber, 2001; Recio&Godino, 2001; Selden&Selden, 2003; Baker&Campbell, 2004; Edwards&Ward,2004; Weber, 2004; Stylianides, Stylianides and Philippou, 2007) indicate that undergraduate students' conception of proof is deficient and they have difficulties in constructing proofs.

Moore (1994) found seven major sources of students difficulties in constructing proofs:

D1: The students did not know the definitions. That is, they were unable to state the definitions.

D2: The students had little intuitive understanding of the concepts.

[1] This paper is the summary of my PhD thesis proposal and I'm at the beginning of the research process.

D3: The students' concept images were inadequate for doing the proofs

D4: The students were unable, or unwilling, to generate and use their own examples.

D5: The students did not know how to use definitions to obtain the overall structure of proofs.

D6: The students were unable to understand and use mathematical language and notation.

D7: The students did not know how to begin proofs.

Gibson (1998) concluded that students' difficulties with mathematical proof are related to the following factors:

1)understanding of the rules and nature of proof

2)conceptual understanding

3)proof techniques and strategies

4)cognitive load

Selden and Selden (2003), described a number of types of reasoning errors and underlying misconceptions. They also classified reasoning errors according to their logical characteristics, that is, according to whether they arise from difficulties in generalization, use of theorems, notation and symbols, nature of proof, or quantification. Some of these errors are beginning with the conclusion, using inverse of a theorem, overextended symbols, weakening the theorem, notational inflezibility, misuse of theorems, ignoring and extending quantifiers etc.

Baker and Campbell (2004) noted that students struggle with understanding the process of proof construction and the precision in writing mathematics. They also found that students demonstrate some misconceptions concerning applying rules of logic to proof construction. Baker and Campbell listed their observations from a transition to proof course as follows:

1) Students struggle with the correct use of logical arguments
2) Students often attempted to write a proof prior to fully evaluating the statement and its implications
3) Students struggle with the precision of mathematical language

Edwards and Ward (2004), investigated the students' use of definitions in constructing proofs and found that many students don't use and categorize mathematical definitions the way mathematicians do, also they can't understand the distinction between everyday language and mathematical language.

Knapp (2005) concluded that students difficulties in constructing proof fit into two categories. First students struggle with the logic, language and culture of the proof as determined by the community. Second students lack the domain specific knowledge, such as definitions, theorems, heuristics and the ability to generate examples.

According to Weber (2006) research deal with cause of students' difficulties with mathematical proof can be categorized into three classes. The first cause of students' difficulties is that they often posses an inaccurate conception of what constitutes a mathematical proof. A second cause is that they don't have an understanding of a theorem or a concept and systematically misapply it. Third reason is that they do not have the decision-making strategies to do so.

Harel and Sowder (2007), suggested a comprehensive perspective that incorporates mathematical, historical-epistemological, cognitive, sociological and instructional factors on the learning and teaching of proof. They state the students' lack of logical maturity and understanding the need for proof as difficulties that can be accounted for by the cognitive factor.

As Harel and Sowder (2007) noted more reserch is needed to systematically document students' difficulties, to understand their nature and their implications for instruction.

However through existing literature, we can organize the main difficulties with proof and some possible causes of these difficulties as follows:

- Perceptions about proof, and students' proof conception and lack of understanding the need for proof (Martin&Harel, 1989; Alibert&Thomas, 1991; Selden&Selden, 2007a; Knuth&Elliot, 1997; Knapp, 2005; Almeida, 2000; Harel&Sowder, 2007; Weber, 2006)
- Students don't know how to begin proof (Moore, 1994; Selden&Selden,2003, 2007a; Baker&Campbell, 2004)
- The lack of understanding and using of mathematical definitions (Edwards&Ward, 2004; Knapp, 2006)
- Inadequate knowledge of mathematical theorems and concepts (Moore, 1994; Hart, 1994; Dreyfus, 1999; Weber, 2006)
- Misuse of mathematical theorems and concepts (Selden&Selden, 2007a; Pedemonte, 2007)
- The lack of logic rules and using of quantifiers (Epp, 2003; Selden&Selden, 2007a; Baker&Campbell, 2004; Harel&Sowder, 2007)
- The inability of follow a chain of reasoning and lack of logical maturity (Dreyfus, 1999; Knapp, 2005; Weber, 2001; Harel&Sowder, 2007)
- Students' knowledge about proof methods is inadequate and they can't apply them correctly (Stylianides, Stylianides, Philippou, 2004, 2007)
- The difference between everyday language and mathematical language and misuse of mathematical language (Moore,1994; Epp, 2003; Ferrari, 2004; Baker&Campbell, 2004; Edwards&Ward, 2004; Selden&Selden, 2007a;)
- The students have difficulties with proof writing and can't express what they think (Dreyfus, 1999; Dubinsky, 2000)

Recent studies emphasize the inadequacy of research that focuse on how students begin the process of constructing a proof and how teaching strategies and learning experiences effect the development of students' understanding. But there is a growing body of work concerning students' attitudes towards proof, proof schemes, and difficulties in constructing correct and valid proofs (Smith, 2006). Harel, Selden and Selden (2006) noted that " We know where the students are, we know where the mathematicians are, but we just don't know how to get tetriary mathematics students from where they are to where we want them to be".

Mariotti (2006) concluded that recent studies on the theme of proof shows a move away from early studies, focussed on students' conceptions of proof and on difficulties with proof and proving, towards studies discussed opinions on whether and how is it possible to overcome such difficulties through appropriate teaching interventions. But there is a need to conduct more studies focussed on what sort of teaching can facilitate students' proof construction.

Purpose of the study

Instruction have a great effect on students' conceptions of proof, proving abilities and difficulties in constructing proof. The latter phase of determining the difficulties with mathematical proof is to study what instructional interventions can be effective in overcoming these difficulties.

The purpose of this study is to determine undergraduate students difficulties with mathematical proof, to design instructional activities for overcoming these difficulties and to investigate the effects of these activities on students' conceptions of proof and proof construction processes. The research questions can be stated as follows:

1)What are the students' difficulties with mathematical proof?

2)What kind of instructional activities can be designed to overcome these difficulties?

At the end of the study it is intended to make suggestions about content and teaching strategies of the advanced mathematics courses if the instructional activities become effective in overcoming students' difficulties.

Methodology

It seems that this study is going to be a design research. To clarify what design research is Gravemeijer and Cobb (2006) discussed the three phase of conducting a design experiment, which are 1) preparing for the experiment 2) experimenting in the classroom 3) conducting retrospective analyses. The purpose of the design experiment is to develop theories about both the process of learning and the means that are designed to support that learning. So that design experiment has a cyclic process of (re)designing, and testing instructional activities and other aspects of the design. The researcher takes the responsibility for the design of the instruction of a classroom for an extended period of time (Gravemeijer&Cobb, 2006).

This study will be conducted with undergraduate students of a secondary school mathematics teacher training program i.e. with preservice mathematics teachers. The students who have taken Abstract Mathematics course, that has similar content with transition to proof courses in many universities, will participate the study. This course includes topics such as propositions, sets, relations and functions, axiom of choice, algebraic structures and number systems also in this course students become familiar with proof methods. We can say that students first meet formal mathematical proofs in this course. The participants will be choosen purposefully according to their lecturers' opinions, among volunteers because it is important for the continuity of their active participation.

This qualitative study has two parts. First the participants' difficulties will be determined and then teaching activities will be designed for overcoming these difficulties. In the first part of the study, the difficulties students cope with in the proving process will be listed through relevant studies in the field and experts' opinions. Then questions will be designed and asked to participants to ascertain their difficulties with proof. This will be a written exam and students' answers will be analysed deeply by the researcher. Also every participant will be interviewed individually to investigate their conceptions of proof and views about proof construction. At the end of the interview a proof construction task will be given to the student and asked to think aloud. Each interview will be video or tape recorded and then transcribed and analysed deeply. The aim of the task given to the students is to clarify whether there are other difficulties that couldn't observed through the written exam.

In the second part of the study, after the frequent errors and difficulties has been determined, teaching activities will be designed with aim of overcoming these difficulties. The study will continue approximately 5–8 weeks and the researcher will also be the teacher. After teaching sessions the students will be asked questions to investigate whether teaching activities become effective in overcoming students' predetermined difficulties. Students' studies and homeworks, classroom observations and lecture notes that collected during the study will be the sources of data. At the end of the study one more structured interview will be held with participants. The purpose of this interview is to determine students' views on instruction and to investigate whether students' conceptions of proof has changed after instruction.

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