READING GEOMETRICAL DIAGRAMS: A SUGGESTED FRAMEWORK

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INTRODUCTION:

In my study, I intend to investigate the role of mathematical visual representations in the construction of mathematical meaning. It has been argued that meaning does not reside only in written and spoken language but also in other different modes of communication such as visual representations, gestures and actions (Kress & van Leeuwen, 2006; Lemke, 1998; Morgan, 1996; O'Halloran, 1999).

Mathematics is a multimodal/multisemiotic discourse where different modes of communication take place such as verbal language, algebraic notations, visual forms and gesture. These different modes may offer different meanings or they may convey one set of meanings (Kress & van Leeuwen, 2006). The verbal language in (mathematical) texts, for instance, despite its power, has limited ability 'to represent spatial relations such as the angles of a triangle (..) or irrational ratios' (Lemke, 1999, p. 175). Thus we need diagrams or algebraic notations to represent these qualities or quantities enabling us to re-examine the argument. In the same manner, gestures help in representing dynamic acts, which both language and visual representations have limited ability to represent these acts. It is the deployment of all these (and other) modes which carries the 'unified' meanings (Lemke, 1999).

Beside the research that has been done concerning the verbal components of mathematical texts (e.g. Morgan, 1996), there is a need to develop tools to describe the other components. This study aims to offer a descriptive framework for geometrical diagrams and to explore how students make sense of them. The main question is *what do geometrical representations contribute to the construction of mathematical meaning?*

BACKGROUND:

A general overview of the status of visual representations, such as diagrams, in mathematical texts indicates that these representations are a) limited in representing knowledge with possible misuse of diagrams (Shin, 1994 as mentioned in O'Halloran,

1999) and b) of an 'informal and personal nature' (Misfeldt, 2007). One main reason for this view is that the main stream among mathematicians conceives mathematics as abstract, formal, impersonal and symbolic (Morgan, 1996). At most, mathematicians consider these representations have messages or meanings which students need to grasp or discover (Shuard & Rothery, 1984). In my prospective study, I consider visual representations as available resources for meaning-making.

Halliday (1985) argues that any text fulfils three functions: ideational, interpersonal, and textual. Our ideas about the world are represented in the ideational meaning, the interpersonal meaning is realised by the relationships constructed with others through communication. The textual meaning is realised as these representations are presented in a coherent way. This descriptive framework is called Systemic Functional Linguistics (SFL). Although this framework was initially developed to account for verbal modes of communication only, it has been extended to include other (non-verbal) modes too. Kress & van Leeuwen (2006), for instance, have developed a grammar to 'read' images using 'representation, interactive and compositional' corresponding to Hallidayan terms respectively. Other examples are: Lemke's studies in science education and language (e.g. 1998), Morgan (1996) and O'Halloran (1999) in mathematics education.

Following the efforts of previous research (Morgan, 1996; O'Halloran, 1999), I intend to investigate what meanings visual representations do offer. As a first step towards this aim, I have developed two iterated versions of a suggested framework. In this paper I use the second developed version.

SOURCES OF DATA: A METHODOLOGY

In order to develop the second version of the framework, I looked at various geometry textbooks and the students' written texts produced for my research. The main sources of data are observation school students in their classes and while they solve problems in groups; teachers' interview and geometry textbooks. In group problem solving I presented two geometrical problems to groups in the class (three students in each) and video- or audio- recorded the students during their discussion and solution. I also conducted teachers' interviews focusing on their views of: students' 'reading' of diagrams, the presentation of diagrams in textbooks and the role of diagrams in the mathematical texts.

ANALYSIS: AN EXAMPLE

The analysis process is still ongoing. The developed framework uses the 'trifunctional' perspective. However, because of the limitation of the space available, I present only the ideational/representational function illustrated by various examples.

The Representational meaning:

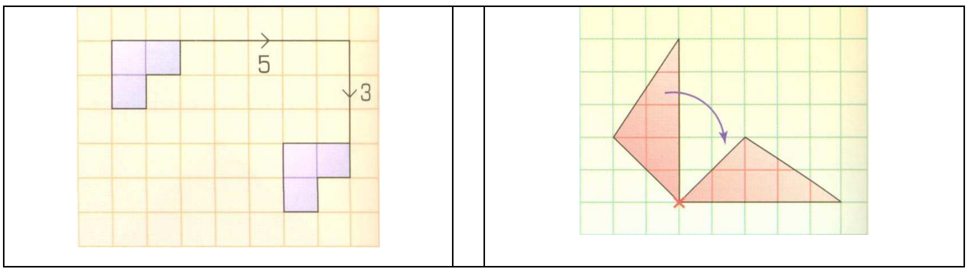
The picture of mathematical activity may be represented through the examination of types of processes and participants acting in them. This meaning is realised by determining the nature of the diagram: in particular, whether it is a narrative structure or conceptual structure.

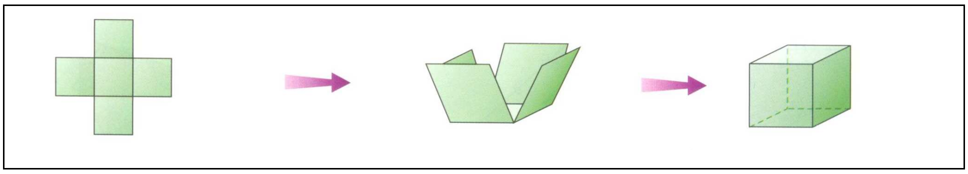
*** Narrative structures:**

Morgan (1996) uses the transitivity system, suggested by Halliday (1985), to analyse the representation of the mathematical activity in any verbal-mathematical text. In images, Kress & van Leeuwen (2006) considered the presence of a vector a distinguishing feature of narrative representation. However, this feature within geometrical diagrams has conventional mathematical meaning (parallelism for example) which does not necessarily express an action. Alternatively, I suggest the presence of a *temporal* factor to be the distinguishing feature of the narrative representation. By temporal factor I mean that there is a representation of the time sequence in drawing the diagram and this timeline can be followed to 'read' or make sense of that sequence. This time is 'seen' or 'observed' in the diagram in different ways. In this sense I distinguish between four types:

(1) <u>Directional structure</u>: the *temporal* factor here is presented by a vector (*arrow*) that should be followed to make sense of the argument, the statement or the solution. In the geometry context, arrows which indicate timeline are of two types:

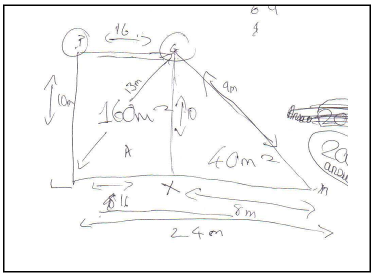
(a) Movement: such as transformation (rotation, reflection and translation, enlargement), sliding, folding, etc.



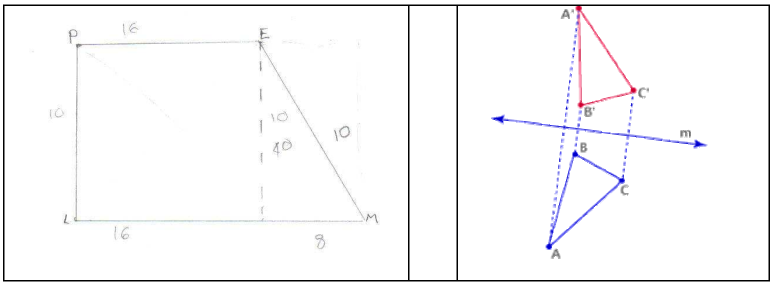


(b) Measurement: to specify a length of a side

The temporality here is realised by the arrows drawn beside the sides in the diagram. These arrows represent measurement as an action or as an ongoing mathematical activity rather than as a property.

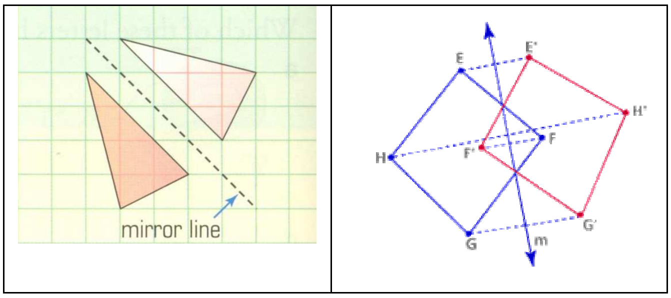


(2) <u>Dotted (dashed) structure</u>: in these structures, the *temporality* is presented by dotted lines and suggests a work has been added to the shape either to solve the problem or to 'show' some features or parts. Again, in the geometry context the dotted structures which imply a timeline are used in different forms. I am interested in the structures that represent actions where a dotted line shows additional work (for example, making a perpendicular line from the vertex of a triangle to its base to calculate the area). Other actions could be: (un)folding, splitting, construction, reflection.



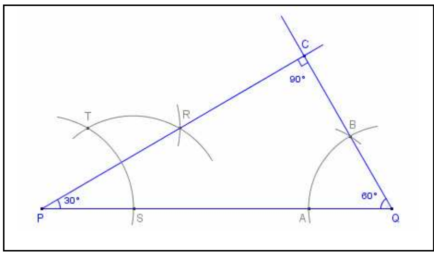
(3) <u>Shaded (coloured) structures</u>:

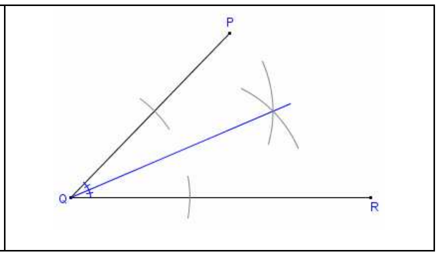
In these structures, the temporality is realised by the presence of the image of the original object.



(4) Construction structures:

The temporality is realised by the construction signs, the common signs are small arcs drawn by the compass, or the 'extra' segments resulted after connecting different points (as at the point C in the first example below)

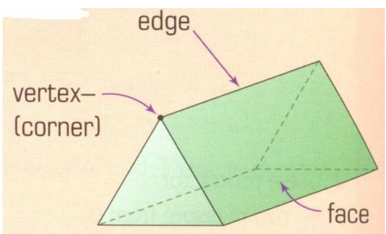


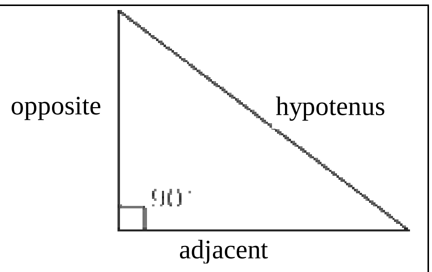


*** Conceptual structures:**

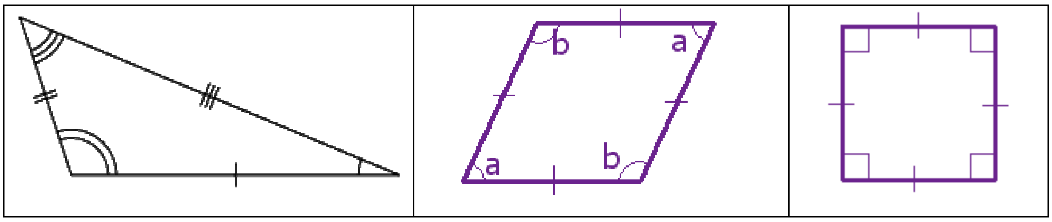
The main feature here is the absence of action where the depicted participants (mathematical objects) stand by themselves. These structures mostly offer information such as definitions, relationships. Kress & van Leeuwen (2006) distinguish three types of conceptual processes: classificational, analytical and symbolic. It is the latter which is I have found applicable in the geometrical diagrams context. Following Halliday (1985), Kress & van Leeuwen (2006) distinguish two types of symbolic structures; Attributive and Suggestive.

1) Symbolic Attributive: In these structures, the identity is given to the depicted participant. In the geometry context, I have found giving names or pointing to specific parts of the diagram are examples of these structures. The diagrams to the right show two cases. The main reason not to consider these arrows in the first example (below) as a directional feature is that these arrows 'come' from outside the diagram itself pointing to specific parts of the diagram.





2) Symbolic Suggestive (Identifying): In these structures, the identity is suggested by the depicted participant itself or, in other words, the participant has these qualities. In the geometry context, definitions of shapes are good examples of these structures.



CONCLUDING REMARKS:

My research is on progress and this framework is still under development. As mentioned earlier and because of the space available I just presented the ideational aspects in that framework. However, there are other aspects within the geometrical diagrams which still need more developing such as, among others, labels in diagrams, the rough-drawn versus the neat-drawn diagrams and the way in which the mathematical text is organised. These aspects are considered within the interpersonal and textual functions. After developing the aimed framework I will use it to analyse the diagrams in the textbooks and students' texts in order to answer my research questions.

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