

Argumentation in School Mathematics

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This study has two aims: (1) to investigate views of mathematics educators regarding deductive reasoning and school mathematics, and (2) to compare and contrast characteristics of argumentation in different algebra classes taught by the same teacher using the same curriculum materials.

The work on the first aim is already in an advanced stage, while we have only begun the analysis of the data related to the second aim. In this paper, and if I get the opportunity to introduce it in the conference and to discuss it with you, I would like to concentrate on the second aim. Therefore, the first aim will be shortly presented here while the second one will be discussed more thoroughly.

1. Views of mathematics educators regarding deductive reasoning and school mathematics

Theoretical Background: Deductive reasoning is most significant in mathematics, especially in the processes of justification and proof. Moreover, it is commonly believed that learning mathematics develops deductive reasoning. Curriculum guidelines, textbooks and teacher guides in many countries state explicitly that one of the goals of mathematics instruction is the development of deductive reasoning that is not restricted to mathematics but can be used in other domains as well (e.g., Qualifications and Curriculum Authority, 2006).

Objectives: This study aimed at examining views and approaches of people involved in mathematics education and logic to deductive reasoning and to the commonly stated goal of using mathematics learning to develop deductive reasoning that is usable outside of mathematical contexts.

Methodology: Data sources include individual semi-structured interviews with 21 people involved in mathematics education and logic (e.g., math teachers, math curriculum developers, researchers in math education). Data analysis is based on the Grounded Theory method.

Findings: Three distinct views regarding the role mathematics learning could play in the development of deductive reasoning were identified (Ayalon and Even, paper to be presented in PME32 conference, July, 2008). Each interviewee's view was interrelated with the interviewee's approach to deductive reasoning and its nature inside and outside mathematics (Ayalon and Even, in press). One group approached deductive reasoning as an action of inference based on formal logic rules, which is usable outside of mathematics alongside "softer" rules of inference. They claimed that there is a need for a deliberate intervention in order for mathematics to develop argumentative habits and skills that do not necessarily comply with rigorous deduction. Another group approached deductive reasoning as an action of inference based on formal logic rules, which is usable in mathematics only. They clearly doubted the possibility of developing deductive reasoning that is not restricted to mathematics. Still, another group regarded learning mathematics as spontaneously improving students' deductive reasoning. Yet, these interviewees addressed only systematic aspects of reasoning that are important to both mathematical and non-mathematical problem-solving situations, with no attention given to the logical validity of the inference.

The second part of our study, which is the main issue of this paper, examines argumentation in the mathematics classroom.

2. Argumentation in the mathematics classroom

As mentioned before, this part of the research is still in an initial stage. Therefore its presentation here will include the following parts only: A brief theoretical background, a definition of the aim, and methodology.

2.1 Theoretical Background

The theoretical background for this study is concerned with two main issues, one is argumentation in the mathematics classroom, and the other is offering mathematics to learners in different classes of the same teacher.

2.1.1 Argumentation in the mathematics classroom

According to Corbett (1986, cited by Voss & van Dyke, 2001), the nature of argumentation has been basically constant over the centuries: Person A states an assertion, and if it is not self-evident, he or she may provide support for it. Or, person B may ask Person A to support it. After A provides the support, B may challenge A's

argument by questioning the accuracy or acceptability of the support, or by questioning whether the stated support is really supportive. Person B may also offer a counter-argument and Person A may try to refute it. Other exchanges may occur that may lead to a resolution or to a "dead-end". This type of argumentative discourse, claims Corbett (1986), reoccurs in different situations, such as children arguing over a toy, political conflicts, debates, arguments presented in courtrooms, and classroom encounters.

Since Plato, argumentation has been considered one of the central means of attaining (or constructing) knowledge. In the last decades, theoretical as well as empirical studies, in mathematics and in other domains of education, examined argumentation as a tool for learning (e.g., Krummheuer, 1995; Kuhn, 1992). These studies claim that argumentative activities such as generating arguments, communicating them to others, and convincing in their truth, require from the participants of the argumentation to deepen and improve their thinking.

Many researchers point at argumentation as a complex process to develop in the classroom and point at the teacher as playing crucial role in carrying it out (e.g., Yackel, 2002). Teachers need to create an atmosphere that encourages practices and norms of argumentation in the classroom. In addition, as the representative of the mathematics community, teachers have to present to the students what constitutes acceptable mathematical arguments.

In recent years, research has begun to identify different types of argumentative discourse in the classroom. For example, Krummeuer (1995) pointed out the reoccurrence of argumentative formats in the mathematics classroom. Atzmon, Hershkowitz, and Schwarz (2006) found that different teachers are characterized by different patterns of encouraging argumentation in the mathematics classroom. These studies indicate the possible existence of patterns of argumentation in the mathematics classroom and the possibility of links between argumentation and the contexts in which it emerges.

2.2.2 Offering mathematics to learners in different classes of the same teacher

In the last decades researchers in mathematics education have begun to study processes of learning and teaching in their natural setting – the classroom, in an attempt to capture their multifaceted. Research shows that different teachers teach

mathematics differently, and that different classrooms are exposed to different experiences of mathematics learning. Research also shows that different teachers enact the same curriculum materials differently. Studying different classes of the same teacher, however, has only now started to be the focus of research studies. The research program *Same Teacher – Different Classes* (Even, plenary address to be presented in PME32 conference, July 2008) belongs to this line of work. Its overarching aim is to gain insights about the interactions among teachers, curriculum, and classrooms. To achieve that the research program compares teaching and learning mathematics in different classes of the same teacher and of different teachers, examining various aspects. One of these aspects is the nature of argumentation developed in the classroom, which the presented study deals with.

2.2 Aim

The aim of the research is to compare and contrast characteristics of argumentation in algebra classes taught by the same teacher using the same curriculum materials.

2.3 Methodology

2.3.1 Research Population

The research comprises of two case studies, each examines two 7th grade classes, from different schools, who study the beginning of algebra with the same teacher using the same curriculum materials (i.e., a total of four classes and two teachers).

2.3.2 Data Collection

Data related to this study was collected by another member of our research group. The primary data source includes video-tapes of the lessons (15-19 lessons in each of the four classes) focused on the teaching of "equivalent algebraic expressions". Another data sources are interviews (conducted by both the observer and me) with the teachers and field notes taken by the observer.

2.3.3 Data Analysis

The data analysis includes several phases, based on Chi's suggested steps of verbal analysis (Chi, 1997):

-*Watching the videos taped in the lessons*; tracing the different parts of the lessons; and identifying situations in which argumentation exists ("argumentation spaces").

-Transcribing selected lessons or parts of lessons. They will include "argumentation spaces". They will be selected from all examined classes, and from various types of algebraic activities (Kieran, 2004). The exact criteria for selection will be decided upon later on.

-Identifying an appropriate unit of analysis. The transcript will be initially segmented according to the discussed mathematics assignments. Then, to identify each new "argumentation unit", I will look for changes in the claim the argument refers to.

-Using known coding schemes and developing a coding scheme. Various options of using known coding schemes (or of combining them) will be examined in order to analyze several aspects of argumentation:

- *The structure of argumentation.* The use of Krummheuer's method (1995) of analyzing collective argumentation will be examined, which in itself is a reduced version of Toulmin's scheme (Toulmin, 1969).
- *The types of justification.* I will examine, for example, the use of Harel's and Sowder's (1998) typology ('external conviction' justification, empirical justification, analytical justification) or Lannin's (2005) classification of students' algebraic justifications.
- *The conceptual basis of argumentation* (the algebraic concepts and principles that the arguments are based on).
- *The contribution of the teacher to the argumentation* (e.g., elements of argumentation the teacher either prompts or provides).
- *The contribution of the students to the argumentation* (e.g., elements of argumentation that students either prompt or provide).
- *The connections between the type of algebraic activity and the nature of argumentation*

Beside these aspects, an open-coding (Strauss, 1987) of the protocols will be used, in order to enable "unexpected" categories to emerge from the data.

-Depicting the coded data and seeking patterns in it.

-Interpreting the patterns.

Also, quantitative tools will be used in order to identify trends in the protocols or to support the suggested findings.

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